Minimax Estimator & empirical Bayes estimator

1. Define: (i) Admissible estimator, (ii) Descision function, (iii) Loss function, (iv) Risk function. [Saleh & Rohatgi: Page-424]

2. Explain the way of obtaining minimax estimator along with example.

3. Let $X \sim b(1,p)$, $p \in \theta = \left(\frac{1}{4}, \frac{1}{2}\right)$ and $A=(a_1, a_2)$. Let the loss function be defined as follows.

	a_1	a_2
$p_1 = \frac{1}{4}$	1	4
$p_2 = \frac{1}{2}$	3	2

Find the minimax solution. [Saleh & Rohatgi: Example 04: Page-425]

4. Define bayesian risk. [Saleh & Rohatgi: Definition 05: Page-426]

5. Let $X \sim b(n, p)$ and $L(p, \delta(x)) = [p - \delta(x)]^2$. Let $\pi(p) = 1$ for $0 < p < 1$ be the a priori PDF of p. Then check whether the bayes estimator is the minimax estimator. [Saleh & Rohatgi: Example 05: Page-428]

6. Let $X \sim N(\mu, 1)$, and let the a priori PDF of μ be $N(0,1)$. Also let $L(p, \delta(x)) = [p - \delta(x)]^2$. Then check whether the bayes estimator is the minimax estimator. [Saleh & Rohatgi: Example 06: Page-428]

7. Let $\{f_\theta : \theta \in \theta\}$ be a family of PDFs (PMFs), and suppose that an estimator δ^* of θ is a Bayes estimator corresponding to an a priori distribution π on θ. If the risk function $R(\theta, \delta^*)$ is constant on θ,

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then δ^* is a minimax estimator for θ. [Saleh & Rohatgi: Theorem 02: Page-435]

8. Let $X \sim b(n, p)$, $0 < p < 1$. We seek a minimax estimator of p of the form $\alpha X + \beta$, using the squared-error loss function. [Saleh & Rohatgi: Example 15: Page-436]

EMPIRICAL BAYES ESTIMATOR

9. What do u mean by empirical Bayes estimator ? Explain with an example . [lehmann-and-george-casella .Page 262]

10. Let $X_k \sim binomial(p_k ,n) and \ p_k \sim beta(a,b)$

$Find \ emperical \ bayes \ estimator \ of \ p_k$.[lehmann-and-george-casella. Page 263.Example 6.2]

11. $If \ X_1 ,X_2 ,.....X_p \ has \ the \ density \ p_\eta(x) = e^{\sum_{i=1}^{k} \eta_i x_i - A(\eta)} h(x) \ and \ \eta \ has \ prior \ density \ \pi(\eta,\lambda), Suppose \ \hat{\lambda}(x) is \ the$

$MLE \ of \ \lambda \ based \ on \ m(x,\lambda). \ Then \ the \ emperical \ bayes \ estimator \ is$

$$E\left(\eta_i, x, \hat{\lambda}\right) = \frac{\delta}{\delta, x_i} \log m\left(x, \hat{\lambda}(x)\right) \qquad \frac{\delta}{\delta, x_i} \log \left(h(x)\right)$$ [lehmann-and-george-casella. Page 265. Theorem : 6.3]

12. Calculate an Empirical Bayes Estimator for the model $X_i|\theta \sim N(\theta,\sigma^2)$, $\Omega \sim N(0,\tau^2)$ [lehmann-and-george-casella. Page 266. Example : 6.4]

13. What is James –Stein estimator ? Write down its properties .[lehmann-and-george-casella. Page 272.]

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14. Find the Bayes risk of the James Stein estimator .[lehmann-and-george-casella . Page 273. Example 7.3]

15. Find the poisson hierarchical Bayes estimator using the following model

$$X_i \sim Poisson(\lambda_i) \quad i=1,2,3......p \ \ independent \ .$$

$\lambda_i \sim Gamma(a,b), \quad i=1,2,3.......p, \ independent \ , \ a \ known$ [lehmann

-and-george-casella .Page : 268, Example:6.6]